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**EXPERIMENTAL AND NUMERIC ANALYSIS OF**  
**SPRING-LOADED PRESSURE RELIEF VALE**  
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## Abstract

The majority of oil and refined-product pipelines in Brazil have their protection systems design based on pressure relief valves. Thus, the proper design and operation of these valves is essential to ensure the safety of pipelines and loading/unloading terminals during any abnormal operation conditions that generate overpressures. These valves work by relieving the internal pressure in case it exceeds a set value. In simple terms, the spring-type pressure relief valve has a disk which is pressed by a spring against the inlet nozzle of the valve. When the pressure rises, the force generated on the surface of the disc increases and, depending on the pressure relief valve set point, the force due to pressure overcomes the force exerted by the spring, causing the disk to rise and discharge the fluid through the outlet nozzle to the relief line, reducing the pressure level within the pipeline. Using this principle, the relief valve ensures that the pipeline is not subjected to high transient pressures, which could, otherwise, lead to pipeline or equipment rupture and possible product leakage. Despite its importance, the models commercially available to simulate the transient behavior of pressure relief valves do not present a satisfactory performance. The present paper presents an experimental study aimed at determining the dynamic behavior of a commercial spring-type relief valve. The valve was installed in a pipe loop where the flow was established. The valve and the loop were instrumented with pressure and flow transducers. The transient motion of the valve disc was measured with a fast-response displacement transducer. The transient in the flow loop was generated by the controlled closing of a block valve positioned downstream of the relief valve. The recorded transient data for disc position, upstream and downstream pressures, and discharge flow rates were compared with results predicted by different models for relief valve dynamic behavior implemented in a commercial pipeline flow simulation software.

## 1. Introduction

Pressure relief valves (PRV) are equipments of fundamental importance to ensure the safe operation of liquid pipelines and loading/unloading facilities. The proper design and operation of this equipment protects life, property and the environment. They are designed to act as the last resource in the pipeline safety procedures.

The majority of Brazilian pipelines and loading/unloading terminals employ PRV's of the spring-loaded type. These are purely mechanical devices which are activated when the internal pressure in the pipeline rises above a pre-determined set point and forces a spring-loaded disc to displace, allowing fluid to flow through the relief line, thereby decreasing the pressure level in the pipeline.

Figure 1 presents an illustration of the expected pressure behavior at a specific position in a liquid pipeline after the sudden blockage of the flow caused by the fast closing of a block valve. After the blockage, the pressure at steady state level,  $P_{st}$ , rises sharply up to the surge pressure value,  $P_{surge}$ . The value at which the pressure stabilizes can be either higher or lower than the surge pressure value, depending on the characteristics of the pumps available in the line and on

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the presence of check valves. In case a PRV is used to protect such a line, its dynamic characteristics should be compatible with the line pressure behavior. If the PRV's set point is located between the steady state and the surge pressure value, the response time of the PRV must be very fast, since the rate of pressure increase is high in this region. For set points above the surge pressure the lower rates of pressure increase that prevail admit PRV's with slower response times.

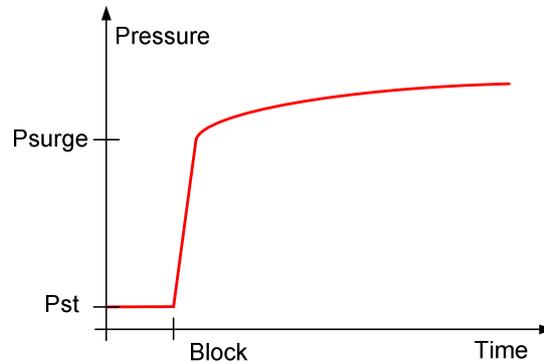


Figure 1 – Typical pressure rise caused by the rapid closing of a pipeline block valve.

As seen by the above exercise, a good understanding of the dynamic behavior of Pressure Relief Valves is of fundamental importance for the proper choice and operation of these equipments. The equation that defines the dimensions of relief valves is specified in the standards ASME section VIII and API 520 [1], and relates the valve geometric characteristics, fluid properties and the square root of the differential pressure. Thus,

$$A = \frac{11,78 Q}{K_d K_w K_c K_v} \sqrt{\frac{G_l}{P_1 - P_2}} \quad (1)$$

Where,

- A is the required effective discharge area, (mm<sup>2</sup>);
- Q is the flow rate, (liters/min);
- K<sub>d</sub> is the coefficient of discharge that should be obtained from the valve manufacturer. For preliminary sizing, an effective discharge coefficient can be used as follows:  
0,65 when a PRV is installed with or without a rupture disk in combination,  
0,62 when a PRV is not installed and sizing is for a rupture disk.
- K<sub>w</sub> is the correction factor due to backpressure. If the backpressure is atmospheric, a value of K<sub>w</sub> equal to 1 should be used.
- K<sub>c</sub> is the combination correction factor for installations with a rupture disk upstream of the PRV. Should be equal to 1, when a rupture disk is not installed, and equal to 0,9, when a rupture disk is installed in combination with a PRV and the combination does not have a certified value.
- K<sub>v</sub> is the correction factor due to viscosity given by  
$$K_v = \left( 0,9935 + \frac{2,878}{Re^{0,5}} + \frac{342,75}{Re^{1,5}} \right)^{-1,0}$$
, *Re* is de Reynolds number.
- G<sub>l</sub> is the specific gravity of the liquid at the flowing temperature referred to water at standard conditions
- P<sub>1</sub> is the upstream relieving pressure, (kPag). This is the set pressure plus allowable overpressure
- P<sub>2</sub> is the backpressure, (kPag).

It should be mentioned that equation (1) is valid for steady state conditions and there is no standard available to guide the specification of relief valves for transient operation conditions. As will be demonstrated in the present work,

the knowledge of the discharge coefficient during the opening of the valve is a key piece of information to allow for the proper prediction of the valve behavior and its impact on the transient pressure in the pipeline. The relationship between the discharge coefficient with the valve opening fraction is dependent on the particular valve design, and it is an information hardly made available by valve manufacturers.

The API 520 standard [1] classifies the pressure relief valves in three different categories, according to its activation method: spring type, pilot operated and others. Figure 2 presents schematic views of spring-type pressure relief valves, the focus of the present work. As seen in the figure, spring-type PRV's are available with or without a bellows. The purpose of the bellows is to isolate the pressure inside the castle from the pressure within the vale, thereby eliminating the effect of the backpressure on the disc and on the pressure set point.

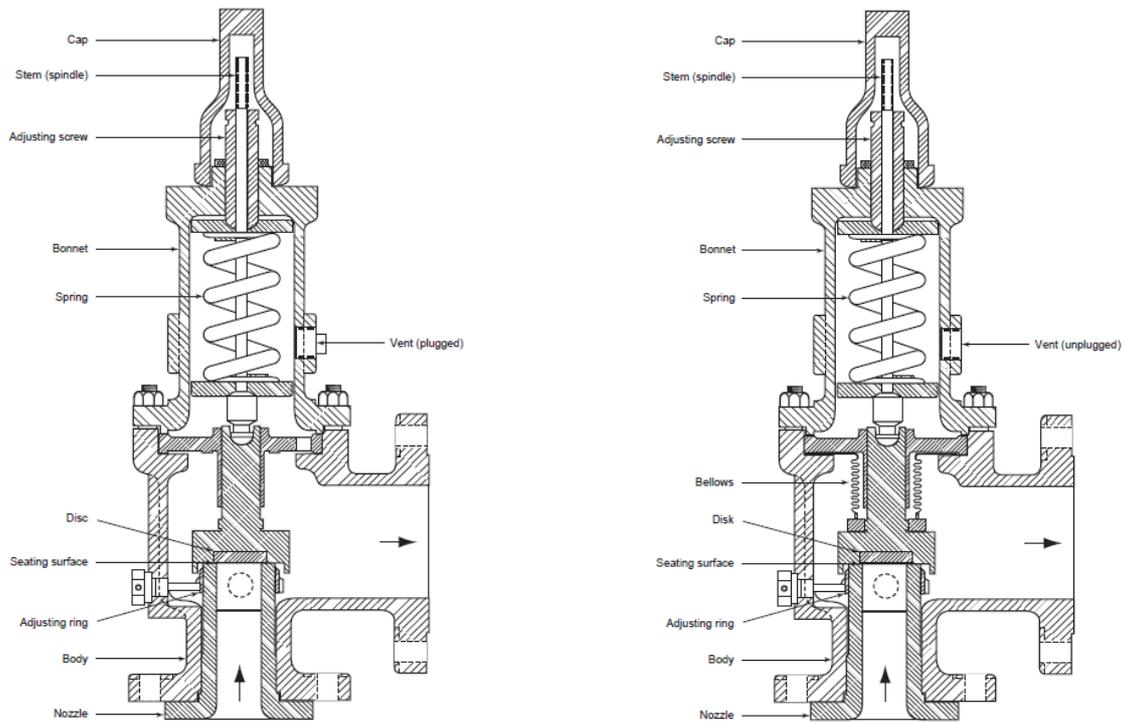


Figure 2 – Pressure Relief Valve with and without bellows (API 520. [1]).

## 2. Transient flow simulation

The dynamic behavior of a Pressure Relief Valve depends not only on the characteristics of the valve itself, but on its interaction with the transient flow in the pipeline system. In the present work the dynamics of the PRV was modeled in conjunction with the transient flow in the test loop used in the experimental part of the work. The commercial software Stoner Pipeline Simulator (SPS) developed by GL Noble Denton, was utilized to model the PRV and the flow loop. The choice of this particular software was based in its wide user base. As other transient pipeline flow simulators, SPS solves the one-dimensional version of the equations representing mass conservation, linear momentum and energy using a finite-difference technique. The SPS software treats PRV's as a dynamic control unit where the transient pressure signal obtained by the simulation at a position just upstream of the PRV is constantly compared with the user-defined valve set point. When the monitored pressure surpasses the set point value, the valve is opened. The dynamic of the valve opening process is controlled by the software by means of two internally defined functions: the valve opening-fraction versus time curve and the valve discharge-coefficient versus opening-fraction curve. The documentation available does not allow the user to have access to the valve dynamic behavior represented by these functions. The user is required to input the valve axial position in the pipeline, the minimum and maximum discharge coefficients, valve flange diameter and valve opening and closing pressures. An option is available for the user to input a discharge coefficient versus opening- fraction curve for the valve. If not input by the user, the software uses the given

maximum and minimum values of the discharge coefficient to internally construct a relationship for the discharge coefficient as a function of valve opening fraction.

Although other valve dynamic models based on basic principles are presently being developed and evaluated [4], the focus of the present work is on a comparison of measured dynamic results for the valve with those predicted by SPS using two different valve models: a model based on the experimentally determined discharge coefficient curve and the model that employs a generic discharge coefficient versus opening fraction curve internally generated by the software, based on the maximum and minimum values of the discharge coefficient input by the user. As will be seen shortly, significant differences were observed on the results predicted by the two valve models. .

### 3. Experiments

Figure 3 presents a schematic view of the test section designed and constructed to conduct the experiments to determine the dynamic behavior of a commercial pressure relief valve. The flow loop was fabricated from 2-inch galvanized steel pipe with a total length of 8 meters. A centrifugal pump was used to circulate water through the loop from an elevated, 150-liters tank. A globe valve was used to control the flow rate in the loop. The spring-loaded pressure relief valve to be studied was installed at a T junction located five meters downstream from the pump. A 2-inch ball valve was installed just downstream of the T junction to block the flow and produce the pressure transient necessary to activate the relief valve. This block valve was motorized in order to allow that its closing time could be performed under controlled conditions. After passing through the block valve the flow returns the main line directly to the tank. A electromagnetic meter was installed in the return line to measure the steady state flow rate. When the flow was blocked and the PRV was activated, the flow was diverted through the PRV to a relief line of the same diameter as the main line. A turbine flow meter positioned in the relief line was used to measure the transient relief flow rate.

Two fast-response pressure transducers were installed upstream and downstream of the PRV to monitor the transient pressure difference across the valve. A displacement transducer of the LVDT type was connected to the shaft of the valve that, in turn, was connected to the valve disc. This transducer furnished the transient position of the disc during the action of the valve. A data acquisition system with high measuring rate was employed to register the pressure, position, and flow rate data.

The PRV tested was dimensioned using the API 520 standard for a flow rate of  $3.7 \text{ m}^3/\text{h}$  at a set point pressure of  $2.0 \text{ kgf/cm}^2$  and a back pressure of  $0.2 \text{ kgf/cm}^2$ . The area of the orifice was equal to  $70.97 \text{ mm}^2$  or  $0.110 \text{ in}^2$ , and the connecting flange was 1-in diameter.

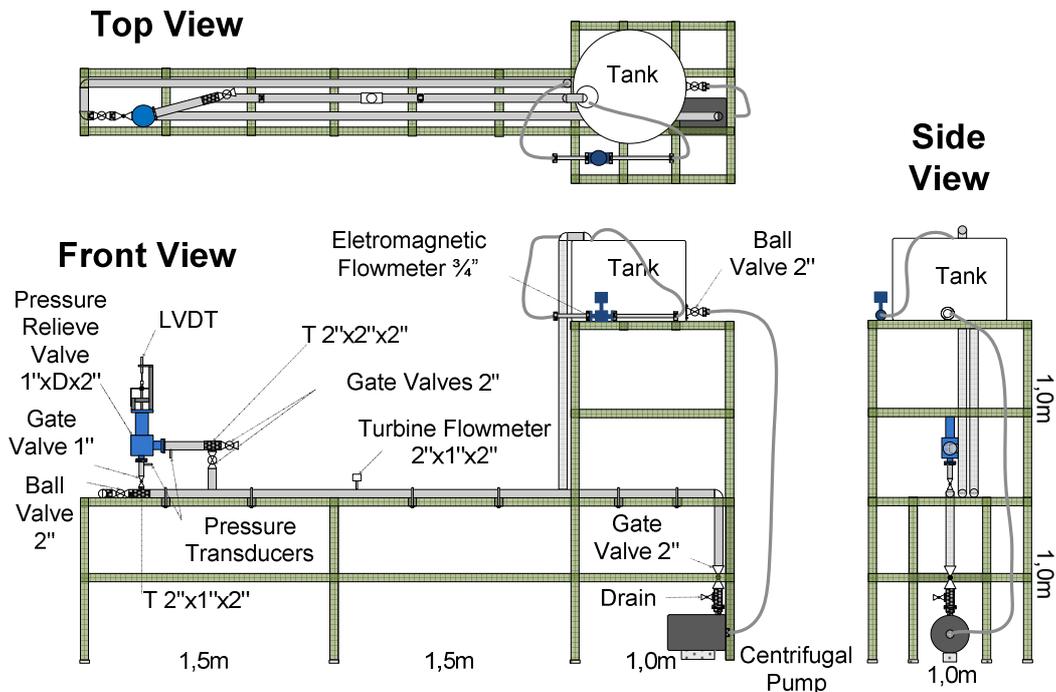


Figure 3 – Schematic view of the experimental test section.

## 4. Results

The results obtained in the present study will be presented in this section. The presentation starts with data displaying the flow characteristics of the pressure relief valve represented by the steady state and transient discharge coefficients, and by the pressure relief set point. Following, a comparison of the measured data with the numerical predictions obtained with the SPS software for the two valve models is presented.

### 4.1 Steady State Discharge Coefficient

The discharge coefficient,  $C_d$ , for the pressure relief valve during steady flow was determined for different flow rates and valve openings. To this end, the shaft connected to the valve disc was fixed at different positions, ranging from fully open to 10% open. For each position, five values of the steady state flow rates were tested, namely, 2, 2.5, 3, 3.5 and 4 m<sup>3</sup>/h. The upstream and downstream pressure values were registered and the discharge coefficient was calculated by using the defining equation (2). In the equation the area,  $A$ , was taken as 70.97 mm<sup>2</sup>, and the water density,  $\rho$ , as 998 kg/m<sup>3</sup>. In the equation,  $Q_s$  is the volumetric flow rate through the valve and,  $P_a$  and  $P_o$  are, respectively, the upstream and downstream pressures measured by the transducers.

$$Q_s = C_d A \sqrt{2 \frac{(P_a - P_o)}{\rho}} \quad (2)$$

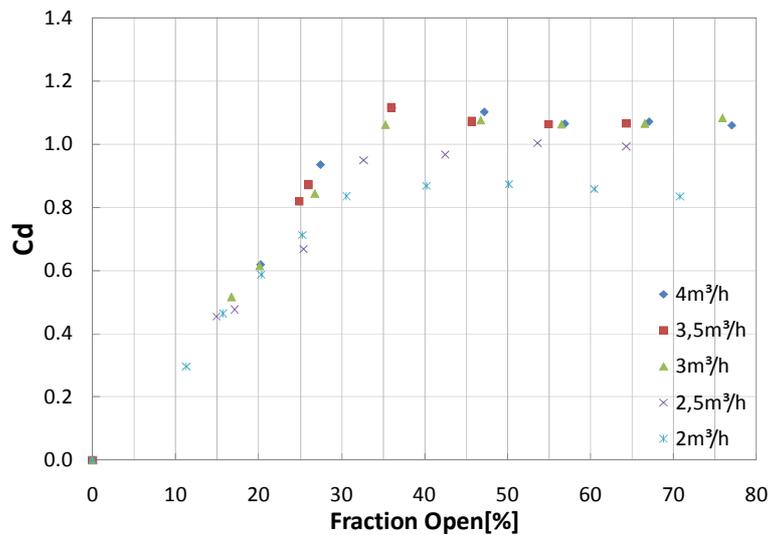


Figure 4 – PRV discharge coefficient as a function of opening fraction, for different flow rates. Steady state condition.

Figure 4 presents the experimental results obtained for the PRV's discharge coefficient as a function of valve opening fraction, for the different flow rates indicated in the figure. The tests were conducted for the steady state regime, as already mentioned. The results presented indicate that a linear dependence between the discharge coefficient and valve opening exists for opening fractions up to 35%. Further, it is seen that in this region of the curve there is practically no dependence on the flow rate. Beyond the 35% opening fraction, the discharge coefficient is seen to level off and an influence of the flow rate can be observed, specially for the lowest value of the flow rate tested. A possible explanation for this dependence of flow rate comes from the fact that the flow over sharp-edged bodies, such as the valve disk, tends to present drag coefficients that are practically insensitive to flow rate (or to Reynolds number) since the flow separation points are normally fixed at the location of the body's sharp edge. The separation points, however, can move along the disc for lower flow rates (or lower values of the Reynolds number) changing the drag offered by the disc what determines the discharge coefficient.

#### 4.2 Transient Discharge Coefficient

The instantaneous data measured for the upstream and downstream pressures, relief flow rates and valve opening fractions allows the determination of the transient valve discharge coefficient. The comparison of the transient and steady state discharge coefficients constitutes valuable information for the dynamic simulation of PRV's.

Although the response time of the pressure and displacement transducers employed was considered adequate for the experiments conducted, the turbine flow meter employed for measuring the relief flow rate displayed a time-response slower than what was considered necessary to resolve the initial stages of the transient flow rate measurements. It was verified by analyzing the measured data that the turbine flow meter indicated a zero reading when the displacement transducer was indicating that the valve was open and the pressure transducers, at the same instant of time, indicated the presence of a pressure difference across the valve. This finding was an indication that the inertia of the turbine meter did not allow it to respond to the initial stages of the transient flow. The observation of the displacement and pressure data indicated that this time delay of the turbine meter was of the order of 40 ms.

Figure 5 presents the results obtained for the transient discharge coefficients associated with the PRV tested. The figure presents the value of the coefficient as a function of the opening fraction for a flow rate of 3.7 m<sup>3</sup>/h. The transient was generated by closing the block valve in 1.5 s. The steady state discharge coefficient is plotted in the figure for comparison purposes. Two transient discharge coefficients are plotted in the figure. The curve to the right refers to the actual data measured in the experiments. The curve to the left was determined by correcting the flow data by the 40 ms time delay mentioned above.

An analysis of the data presented in Figure 5 leads to useful information. The remarkable agreement observed between the transient and steady state values of the discharge coefficient after the time delay correction is applied is an indication that the steady state data can be used to model the dynamic behavior of pressure relief valves. Experiments for the determination of steady state values of discharge coefficients are much easier to conduct and require less expensive equipment. These findings need to be supported by additional experimental data for other flow rate values and for more severe transients generated by a faster closing of the block valve. These experiments are presently being conducted in our laboratory.

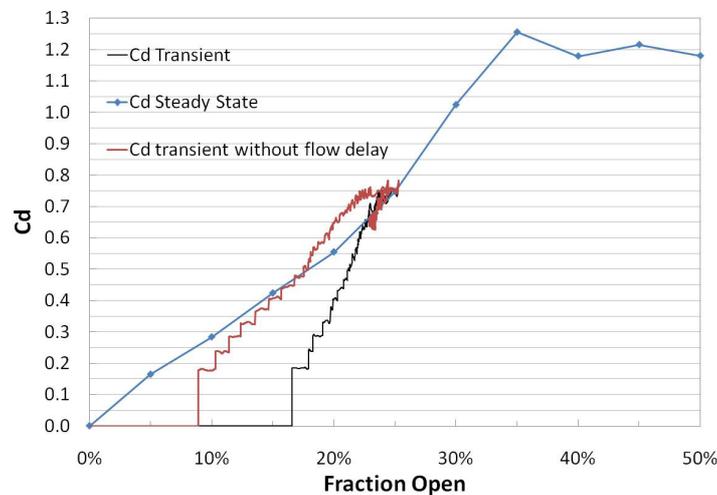


Figure 5 – Comparison between PRV discharge coefficients for steady state and transient flow conditions, for flow rate of 3.7 m<sup>3</sup>/h and block valve closing time of 1.5 s.

#### 4.3 Valve Pressure Relief Set Point

The pressure level at which the PRV opens, the set point, is a relevant information to characterize its dynamics behavior. During the tests conducted, the pressure at which the valve opened in response to a fast closing of the block valve was determined through the analysis of the recorded transient data for the upstream pressure and valve opening fraction. In the experiments the valve was considered open when the opening fraction reached a value of 0.005 or 0.5%. At this opening, the upstream pressure was recorded and defined as the opening pressure.

Table 1 presents the results obtained for the opening pressure for two values of the flow rate, and for different values of the blocking time which characterizes the intensity of the transient generated. Each experiment was conducted twice, as indicated in the table.

Table 1 – PRV Opening Pressures.

Initial Flow	Blocking time	Opening Pressure Test 1	Opening Pressure Test 2
<b>m<sup>3</sup>/h</b>	<b>s</b>	<b>kgf/cm<sup>2</sup></b>	<b>kgf/cm<sup>2</sup></b>
4,5	0,2	2,44	2,49
	0,3	2,42	2,45
	0,5	2,41	2,35
	0,8	2,31	2,40
	1,0	2,23	2,20
	1,5	2,26	2,23
5,9	0,2	2,38	2,46
	0,3	2,41	2,43
	0,5	2,31	2,35
	0,8	2,33	2,31
	1,0	2,22	2,24
	1,5	2,25	2,23
<b>Average Opening Pressure</b>		<b>2,34</b>	<b>kgf/cm<sup>2</sup></b>

The results displayed in Table 1 were averaged yielding an average value for the opening pressure of 2.34 kgf/cm<sup>2</sup>. It is interesting to note that this value differs from the opening pressure stated in the valve's certificate issued by the manufacturer, which is equal to 2.07 kgf/cm<sup>2</sup>. It is believed that this difference might be related to the calibration method employed by the manufacturer that, possible, is based on a static experiment. This issue deserves further investigation.

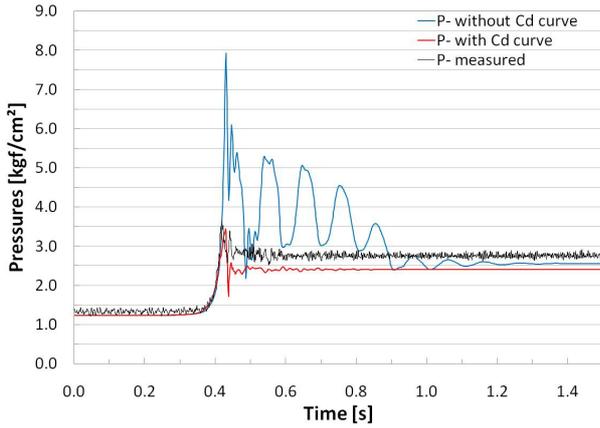
#### 4.4 Comparison of Experimental and Simulation Results

This section presents the main results of this study, namely, the comparisons of the dynamic behavior of the tested PRV, obtained from the experiments and that provided by the numerical simulations. The comparisons are presented in Figures 6 to 8, respectively for upstream pressure, relief flow rate, and valve opening fraction. In each figure, three curves are presented. One of them represents the experimental data obtained from the tests conducted. The two other curves are numerical predictions obtained employing the SPS software using different simulation models for the PRV dynamics. One of them makes use of the discharge coefficient curve internally generated by the software, calculated with the maximum and minimum values input to the program and assuming a linear dependence between the coefficient and the valve opening fraction. The second model employs an experimentally determined curve for the discharge coefficient as a function of valve opening, such as that presented in Figure 4.

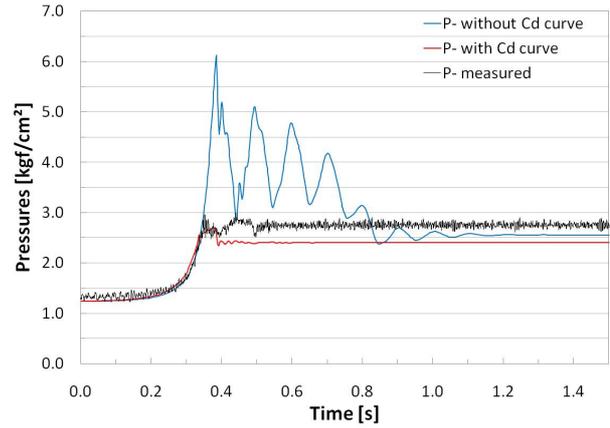
Figure 6 shows the comparison between the numerically and experimentally determined pressures upstream of the relief valve,  $P_-$ . The data corresponds to an initial flow rate of 5.9 m<sup>3</sup> / h, which provides a relief flow rate equal to 3.7 m<sup>3</sup> / h. Different closing times of the block valve are presented, as indicated in the legends of Figures 6(a) to 6(d), namely, 0.2 s, 0.5 s, 1.0 s and 1.5 s.

An observation of the experimental data displayed in Figure 6(a) shows that the upstream pressure value is constant up to the time when the block valve starts to close. The pressure is seen to increase sharply up to 3.5 kgf/cm<sup>2</sup> when the PRV opens and the pressures levels off at about 2.75 kgf/cm<sup>2</sup>. The pressure level after stabilization is higher than that prior to the closing of the valve due to the fact that, after the blockage, all the flow passes through the PRV, which presents a higher pressure drop. A significant difference is observed between the measured results and those predicted by the simulation employing the internally determined discharge coefficient (labeled as "without  $C_d$  curve" in the figure). A much better agreement with experiments is obtained with the use of the experimentally determined curve for the discharge coefficient (labeled "with  $C_d$  curve" in the figure). The oscillations found in the predictions of the first model are attributed to the larger openings produced by the internal valve dynamic model that provides less dumping to the waves traveling in the relief line. The poor agreement is seen to prevail even for the less severe transients imposed by the slower closing of the block valve.

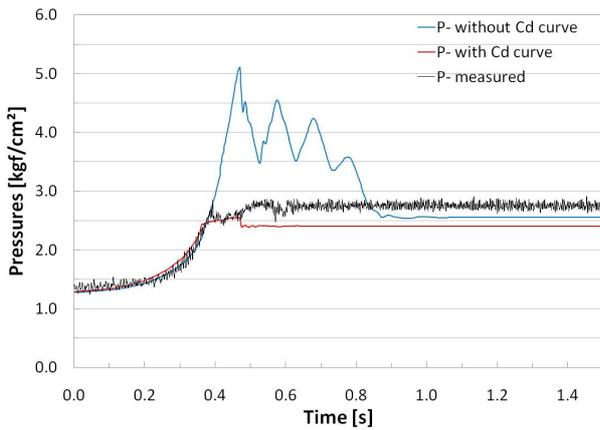
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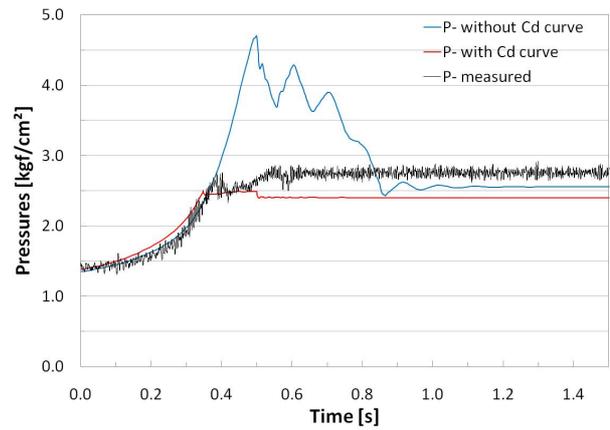
(a) Blockage in 0,2s



(b) Blockage in 0,5s



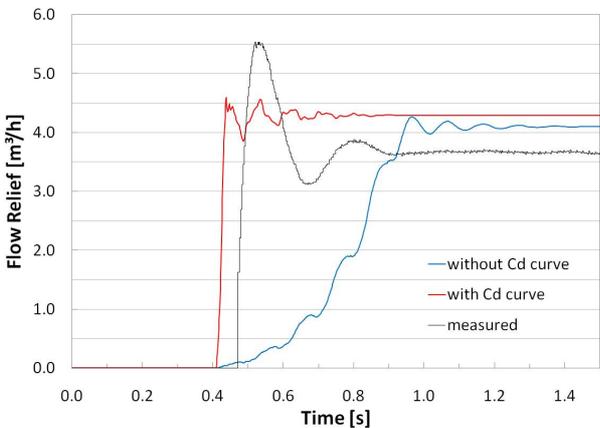
(c) Blockage in 1,0s



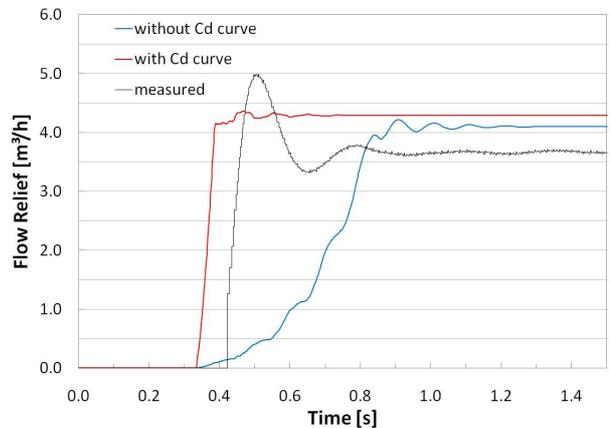
(d) Blockage in 1,5s

Figure 6 – Measured and predicted upstream pressure values for initial flow rate of 5,9m<sup>3</sup>/h and different block valve closing times.

Figure 7 shows the comparison between measured and predicted relief flow rates, for the same flow conditions and block valve closing time as those of the previous figure. It can be seen in the figure that the incorporation of the measured discharge coefficient in the valve dynamic model produces reasonable agreement with the experimental data. In fact, the model that does not include the actual discharge coefficient data is not capable to predict even the qualitative trends of the relief flow rate behavior. The time lag verified in the surge flow from experiments and SPS simulations can be attributed to the inertia of the turbine flow meter used to measure flow through the relief valve, as already commented.

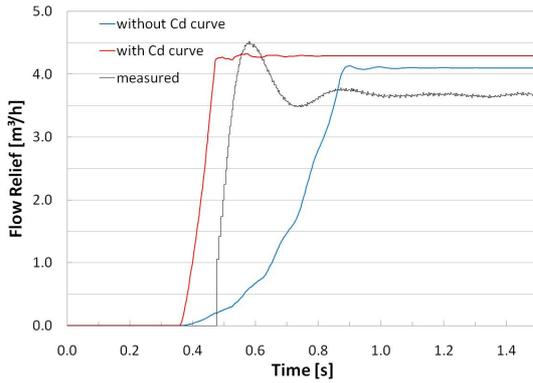


(a) Blockage in 0,2s

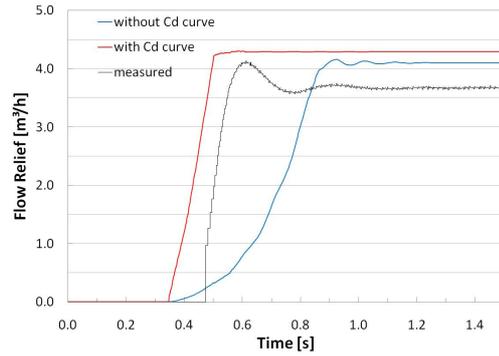


(b) Blockage in 0,5s

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(c) Blockage in 1,0s

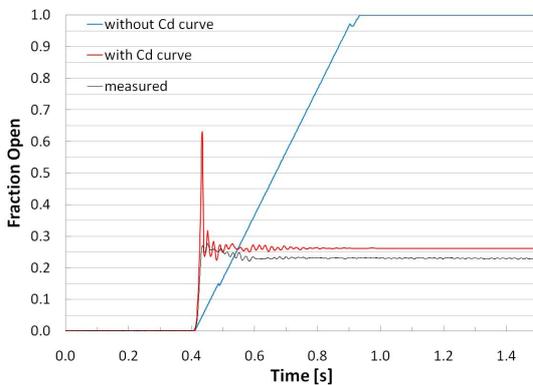


(d) Blockage in 1,5s

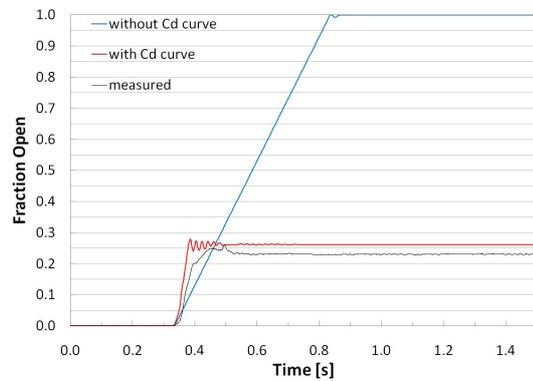
Figure 7 – Measured and predicted relief flow rates for initial flow rate of  $5,9\text{m}^3/\text{h}$  and different block valve closing times.

Finally, Figure 8 presents a comparison of measured and predicted valve opening fractions for the same flow conditions and block valve times as those of the previous figures. Again, the model that incorporates the actual discharge coefficient data predicts quite well the dynamics of the valve opening. For the faster block valve test, Figure 8(a), the model over predicts the initial values of the opening fraction. On the other hand, the model that does not consider the actual discharge coefficient data assuming a linear dependence between discharge coefficient and opening fraction, presents totally unrealistic results.

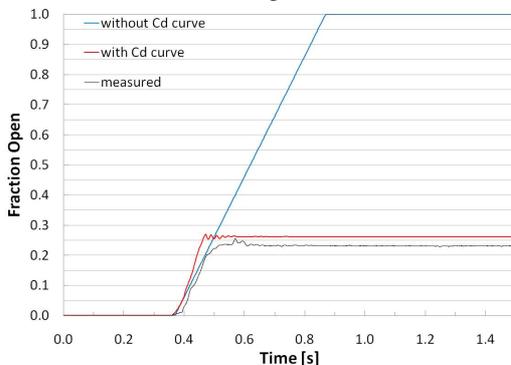
These edicts closing time shows the comparative of opening fraction for the same cases. Again, it was verify that the model without the experimental  $C_d$  curve is unable to predict the qualitative behavior of the opening fraction providing different results from those measured. The model with the curve shows good agreement with experiment. Analysis of these results reinforces the observation checked against the pressure behavior that emphasizes the need to include in the model of the valve the relation between the discharge coefficients and the opening fraction.



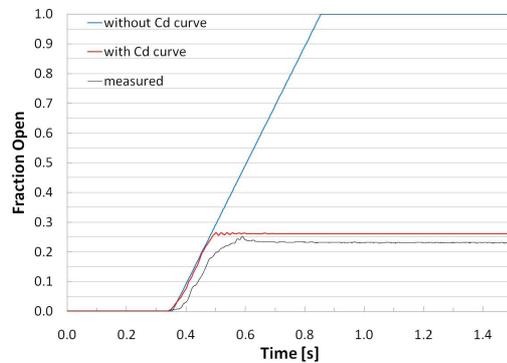
(a) Blockage in 0,2s



(b) Blockage in 0,5s



(c) Blockage in 1,0s



(d) Blockage in 1,5s

Figure 8 – Measured and predicted valve opening fractions for initial flow rate of  $5,9\text{m}^3/\text{h}$  and different block valve closing times.

## 5. Concluding Remarks

The present paper studied the dynamic behavior of a spring-type pressure relief valve. In the experimental part of the work a commercial valve was mounted in a flow loop where transient flows of different intensities could be imposed by controlling the closing time of a block valve positioned downstream of the pressure relief valve tested. The valve and the flow loop were instrumented so as to allow the measurement of relevant transient quantities, namely, pressure difference across the valve, relief flow rate and valve opening fraction. A commercially available software was employed for simulating the transient pipeline flow in the test loop, including the pressure relief valve dynamics. The software offers to the users a generic model to simulate the valve behavior, where a control strategy is used to sense the transient pressure level in the pipeline, compare it with the user input valve pressure set point and activate the valve opening and control its dynamic behavior. The generic valve model available utilizes a linear relationship between valve discharge coefficient and valve opening fraction. Alternatively, the software allows for the input of a experimentally determined relationship between valve opening and discharge coefficient.

Experimental results were obtained for the valve discharge coefficient as a function of the valve opening fraction for steady state and transient flow conditions. For the steady state measurements, the valve disc was fixed in position while the pressure difference across the valve and the flow rate were measured. For the transient measurements, the instantaneous data recorded for pressure difference across the valve and valve opening fraction, measured by a fast displacement transducer, were employed in the discharge coefficient calculations. A comparison of the steady state and transient discharge coefficients displayed are remarkable good agreement. This is an important finding that allows the utilization of discharge coefficients measured in steady state conditions in transient applications. Steady state measurements of discharge coefficients are easier and cheaper to perform. More severe transient flow conditions should be tested in order to confirm this finding.

Pressure relief set points were measured for the valve tested, employing the transient data acquired for valve opening fractions and upstream pressures. The values obtained were 13% above the value stated in the valve certificate issued by the manufacturer. This difference might be attributed to a probable static measuring procedure employed by the manufacturer. If, indeed, this is the case, a recommendation for changing the static calibration procedure could be appropriate. This issue should be further investigated.

Comparison of the experimental result and the numerical prediction for upstream pressure, relief flow rate and valve opening fraction as a function of time were performed for different initial flow rates and transient intensities characterized by the different block valve closing times tested. The comparisons indicated that the simulation that employ the generic linear relationship between valve discharge coefficient and opening fraction fail to predict the flow and valve dynamic behavior. In fact, this model was not able to predict the qualitative behavior of the relief valve. The model which uses the experimentally determined discharge coefficient versus valve opening, on the other hand, present a satisfactory performance.

The importance of the availability of accurate information on the discharge coefficient of pressure relief valves was demonstrated in the present work. Predictions of pressure transients in the pipeline that significantly differ from the actual values can be obtained from poor discharge coefficient information. It is recommended that this issue be discussed further and that valve certificates include more detailed information on discharge coefficients and its dependence on valve opening fraction.

## 6. Acknowledgements

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